JONES

Determination of e/m and v for Cathode Rays by the Photographic Method

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# DETERMINATION OF $e/m$ AND $v$ FOR CATHODE RAYS BY THE PHOTOGRAPHIC METHOD

BY

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## THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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# DETERMINATION OF e/m AND v FOR CATHODE RAYS BY THE PHOTOGRAPHIC METHOD.

## I.



## II.



## III.

VII.

# DETERMINATION OF e/m and v FOR CATHODE RAYS BY THE PHOTOGRAPHIC METHOD.

## I.

## PHENOMENA OF DISCHARGE THROUGH A VACUUM TUBE.

Up to the time of Crookes in the early seventies, the phenomena of the discharge of electricity through a vacuum tube were shrouded in darkness and it had come to be regarded as a veritable "theory trap". Since the time of Crookes, however, the amount of work done on the discharge of electricity through gases has increased so steadily and so rapidly that now, as Clerk Maxwell predicted, it is shedding light upon the whole domain of electrical science.

If a glass tube a meter long and three or more centimeters in diameter, with an electrode at each end, is filled with air and connected to an air pump - the discharge tube having a spark gap of two or three centimeters in parallel with it - it will be noticed that for some time after the pump is started the spark leaps across the gap in preference to the longer path through the tube. As the pressure lowers a stage is reached where the spark across the gap ceases and the discharge takes place through the tube as a long thin spark of pinkish color. As the

pressure lowers further the spark splits up into several and at a pressure of a centimeter or less it takes the form of a glow filling the whole tube. Soon this glow is seen to lose its pinkish color, becomes more bluish and splits up into transverse sections not unlike discs seen edgewise, with darker spaces intervening. The positive column which has broken up into these striae now fills almost the whole tube. The anode is now illuminated at only a point or two and the striae extend from near the anode to almost the cathode; the positive column and the cathode being separated by a dark space called the Faraday.dark space. The cathode is completely covered with a velvety glow which, as the exhaustion proceeds, is seen to separate from it and leave between them a very dark region called the Crookes dark space. As the pressure is further reduced the striae disappear and form a continuous glow which is continually pushed along toward the anode by the enlarging Crookes dark space. Soon the dark space extends two centimeters or more from the cathode and a new phenomenon makes its appearance. The dark space has extended to the walls of the tube and these have begun to glow with a brilliant green phosphorescence, evidently the result of being the boundary of the dark space. The color of the discharge through the tube depends upon

the nature of the gas contained in it and the color of the phosphorescence upon the kind of glass used. The tube may be exhausted further until the phosphorescence extends even beyond the anode. The phosphorescence is then seemingly due to a something being shot off from the cathode and striking the walls of the tube; though whatever it is it is invisible until the walls of the tube stop it.

If instead of the tube with the electrodes at the ends one is substituted with the cathode a plate perpendicular to the axis of the tube and the anode a small plate with a hole in its center and placed near the center of the tube, also with its plane perpendicular to the axis of the tube, the anode plate is seen to cast a shadow, with a bright central spot, on the end of the tube. This bright central spot is in direct line with the hole in the anode and the cathode, and the shadow cast by the anode plate is of the same diameter as the plate itself, seeming to show that the cathode rays are shot off at right angles to the cathode surface and travel in straight lines.

## II.

## PROPERTIES OF CATHODE RAYS.

If the plane cathode is replaced by a curved cathode of uniform radius of curvature and a thin piece of platinum is placed at its center of curvature, then under the bombardment of the cathode rays the platinum will glow red and, if the discharge be strong enough, will even become white hot. Cathode rays then possess energy and on being stopped part, at least, of this energy is transformed into heat. If a small "paddle wheel" is placed in the path of the cathode rays so that the rays strike the paddles on one side of the axis the wheel will turn, showing that the rays possess kinetic energy and must accordingly possess mass.

If a magnet is brought near the tube and its stream of cathode rays the beam will be deflected, showing that the rays consist of electrically charged particles in motion. Professor Rowland[1] has shown that an electric charge in motion is equivalent to a current, and from the direction of the deflection of the beam by the magnet and the fact that the spot is not split up into two spots the conclusion is drawn that the cathode

1 Rowland's Papers, p. 128.

rays are charged particles in motion, and it remains to be shown whether the charge is positive or negative. Perrin[1] devised an experiment to show this.

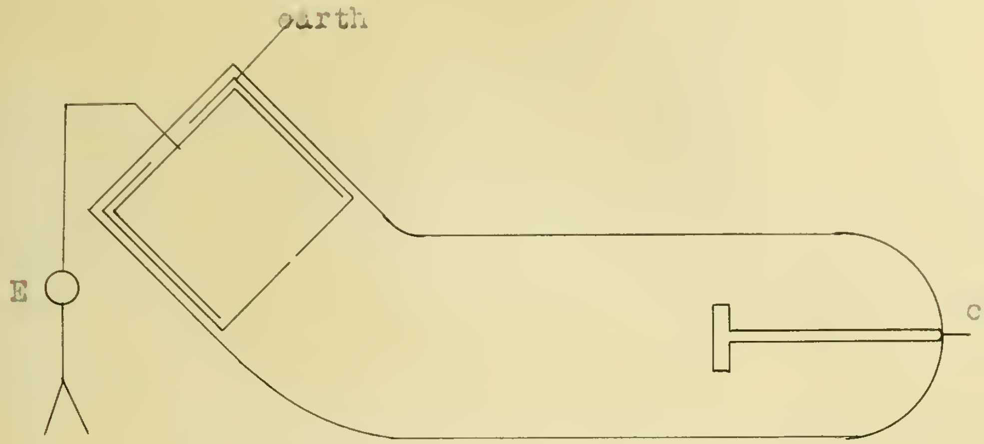



Fig.1.

In a discharge tube like that shown in Fig.1 were placed a pair of Faraday cylinders, the outer one connected to earth and the inner one to an electroscope E. The rays were deflected by a magnet so that they entered the opening in the outer cylinder and struck the inner one. The electroscope at once showed the presence of a charge, found to be negative; thus showing the cathode rays to be negatively charged particles in motion.

When the discharge is passed through a vacuum

1 Compte Rendus, v. 121, p. 1130, 1895.

tube positive charges are formed at the edge of the dark space and fall toward the cathode. During their fall they acquire a velocity, due to the potential gradient, and on striking the cathode give up their charges and give rise to the cathode rays which travel from the cathode in straight lines. The velocity of the cathode particles is due also to the potential gradient, and by the time they travel a few centimeters the velocity acquired is sufficient to produce ions, and thus the boundary of the dark space is marked by the ionized gas.

The velocity of the cathode rays and of the ratio e/m, the charge to the mass, is independent[1] of the nature of the gas in the tube, and within certain limits, of the nature of the electrodes.

Sir J. J. Thomson has shown[2] that when an electric charge in motion is suddenly stopped an electromagnetic pulse is produced. The cathode particles when suddenly stopped, as by the striking against a metal plate placed in their path or against the walls of the discharge tube, give rise to the pulses known as X Rays or Roentgen rays.

If the pressure of the gas is not exceed-

1 J. J. Thomson, Phil. Mag., v. 44, p. 293, 1897.

2 J. J. Thomson, Cond. t. Gases, sec. ed., p. 657.

ingly low the velocity of the rays will be reduced somewhat by collisions with the molecules of gas in the tube.

# III.

## VELOCITY OF CATHODE RAYS IN A GAS OF A CONSIDERABLE PRESSURE.

In measuring the velocity of the cathode rays at a distance from the cathode it has been assumed that the velocity is not appreciably diminished by collisions of the particles with the molecules of the gas. In order that this assumption be valid it is necessary that the pressure of the gas be very low.[1] If the pressure is not sufficiently low the velocity of the particle after traveling a distance x will be smaller and will be, let us say, $v\varepsilon^{-kx}$. Then since

$$\frac{mv^2}{r} = Hev,$$

we may replace v by $v\varepsilon^{-kx}$ and $1/r$ by $\frac{d^2z}{dx^2}$ and get

$$\frac{d^2z}{dx^2} = \frac{He}{mv}\varepsilon^{kx}.$$

If the magnetic field is uniform

$$\frac{dz}{dx} = \frac{He}{mvk}(\varepsilon^{kx} - 1)$$

and,

$$z = \frac{He}{mvk}\left[\frac{\varepsilon^{kx} - 1}{k} - x\right],$$

1. J. J. Thomson, Cond. t. Gases, sec. ed., p. 120.

where Z is the magnetic deflection, say at the outer end of the electrostatic plates, x being the length of the plates. Then since $x = d$, the length of the electrostatic plates we have

$$Z = \frac{He}{mvk} \left[ \frac{\varepsilon^{kd} - 1}{k} - d \right].$$

If Xe is the force on the particle due to the electrostatic field and Y is the deflection at the end of the electrostatic plates due to this field, then,

$$\frac{d^2y}{dt^2} = \frac{Xe}{m}.$$

Integrating, $$\frac{dy}{dt} = \frac{Xe}{m} t,$$

and, $$Y = \frac{Xe}{2m} t^2.$$

But, $$\frac{dx}{dt} = v\,\varepsilon^{-kx}.$$

Rearranging, $$\varepsilon^{kx} dx = v\,dt.$$

Integrating, $$t = \frac{1}{kv} (\varepsilon^{kx} - 1).$$

Substituting this value of t in the equation giving the value of Y and we get,

$$Y = \frac{Xe}{2m} \frac{(\varepsilon^{kd} - 1)^2}{k^2 v^2}.$$

Then, $$\frac{Z^2}{Y} = \frac{H^2ed^2}{2mX}\left[\frac{\varepsilon^{kd}-1}{k} - d\right]\left[\frac{4}{(\varepsilon^{kd}-1)^2\ d^2}\right].$$

If kd is small it is seen that in this case the ratio m/e will be too large and the value of e/m consequently too small.

Sir J. J. Thomson[1] gives the value of k for particles moving with a velocity of about $3 \times 10^9$ cm./sec. as .0085. Thus the pressure of the gas in the discharge tube must be less than .001 cm. if the true velocity is to be measured. If the gas pressure in the tube is considerable it will tend to make the electrical force between the electrostatic plates less than $\frac{V}{d}$ when V is the difference of potential of the plates and d the distance between them. If, in the above equation, X is replaced by V/d the value of e/m would probably be too large if any gas remained in the tube.

1 J. J. Thomson, Cond. t. Gases, sec. ed., p. 121.

# IV.

## METHODS USED TO DETERMINE e/m AND v.

### Schuster's[1] Maximum and Minimum Method for the Values of e/m and v.

If V is the difference of potential existing between the cathode and the screen, the work done in giving the cathode particle its velocity cannot be less than the kinetic energy of the particle. Since Ve is the maximum amount of work that can be done on the particle we have,

$$\frac{1}{2}mv^2 \not> Ve.$$

Schuster measured the radius of curvature of the path of a cathode beam in a magnetic field and found

$$\frac{mv^2}{r} = Hev$$

where r is the radius of curvature of the path. From these two equations he derived

$$\frac{e}{m} \not> \frac{2V}{Hr},$$

which is the maximum value of e/m.

1 Schuster, Proc. Roy. Soc., XLVII., p. 526, 1890.

To find a lower limit Schuster took V equal to the velocity of mean square of the atoms of the gas in the discharge tube. Calling this velocity U

$$\frac{e}{m} \nless \frac{2U}{Hr} .$$

For air Schuster found by this method

$$e/m \ngtr 11 \times 10^5$$
$$e/m \nless 10^3$$

Kaufmann[1] and Simon[2] modified Schuster's method by assuming that the kinetic energy of the particle was the energy gained by a fall through the potential difference between the anode and the cathode. Kaufmann found e/m to be $1.86 \times 10^7$.

Simon made a very large number of experiments with the potential difference varying from 4860 to 11840 volts and found e/m to be $1.865 \times 10^7$. Simon used a Wimshurst machine for the discharge and found the value of e/m to be independent of the potential difference.

Sir J. J. Thomson has pointed out[3] that the

1 Kaufmann, Wied. Ann., v, 61, p. 544, 1897; 62, p. 596, 1897; 65, p. 431, 1898.

2 Simon, Wied. Ann., v. 69, p. 589, 1899.

3 J. J. Thomson, Cond. t. Gases, sec. ed., p. 128.

assumption would tend to make the values of $e/m$ too large as they assume the rays to originate on the cathode itself and that no energy is used in tearing the particle from the cathode.

The fall of potential at the cathode, the electrostatic deflection (over the whole range) and the heating effect, all depend on the value of the kinetic energy. Seitz[1] measured the kinetic energy by all these methods and found it to be the same for all, when the pressure is very low, and equal to $Ve$.

## Lenard's Method.

Lenard[2] used the tendency of a second electrical field to accelerate or retard the motion of the particles.

The rays after leaving the cathode are cut down to a very small pencil and then pass through small openings in the condenser plates $C_1$ and $C_2$, arranged as shown in Fig. 2; of these $C_1$ is always kept connected to earth and $C_2$ may be charged positively or negatively by an electric machine. After leaving the condenser the rays pass between

1 Seitz, Ann. d. Phys., VIII., p. 233, 1902.

2 Lenard, Wied. Ann., XLV., p. 504, 1898.

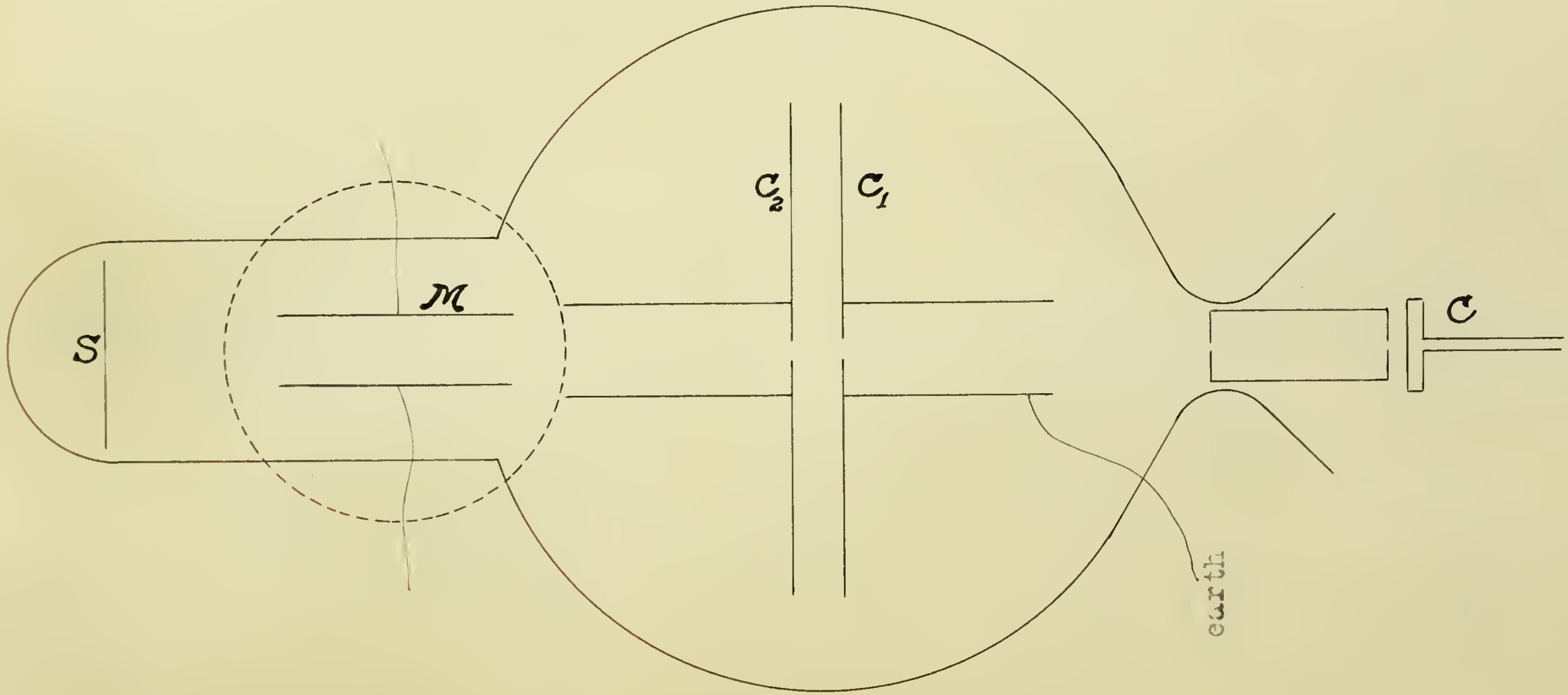



*Fig. 2.*

two electrostatic plates M used to produce the deflection on the screen S. A solenoid, shown by the dotted circle, was used to produce the magnetic deflection. Lenard found that if $C_2$ was charged negatively the velocity was decreased while it was increased if $C_2$ was positively charged.

If the velocity of the rays is $v_1$ when the plates $C_1$ and $C_2$ are at the same potential and $v_2$ when the plate $C_2$ is at a potential V, then, assuming that the whole change in energy is due to the electrical field, we have

$$Ve = \tfrac{1}{2}m(v_2^2 - v_1^2).$$

Calculating from this equation Lenard found the value of e/m to be about $6.6 \times 10^6$.

Becher[1] in 1905, used Lenard's method and found the value of e/m to be $1.8 \times 10^7$.

## Determination of e/m and v by the Heating Effect of the Rays and the Magnetic Deflection.

Sir J. J. Thomson[2] used the method of measuring the kinetic energy of the rays by the heat given up by

1 Becher, Ann. d. Phys., 17, p. 381, 1905.

2 J. J. Thomson, Phil. Mag., v. 44, p. 293, 1897.

them to a thermo couple.

If N is the number of particles striking the thermo couple in unit time then,

$$Q = \tfrac{1}{2}N\, mv^2,$$

where Q is the heat developed in unit time at the thermo couple, assuming that all the energy is transformed into heat.

If the beam is allowed to strike a cylinder connected to an electrometer the rate of increase of negative electricity may be measured.

Then, $$Ne = E.$$

Eliminating N from the two equations we get

$$\frac{1}{2}\,\frac{mv^2}{e} = \frac{Q}{E}$$

The quantity mv/e may be determined from the magnetic deflection and the values of e/m and v may then be deduced. J. J. Thomson found e/m to be $1.7 \times 10^7$.

Wiechert's[1] Maximum and Minimum Values of e/m and v.

1 Wiechert, Sitzungsber. d. Physikal. okonom. Gesell. zu Königsberg, 38, p. 1, 1897.

Wiechert obtained a value for mv/e from the magnetic deflection, and for a second relation put

$$\frac{1}{2}\frac{mv^2}{e} = KV$$

where V is the difference of potential between the electrodes in the discharge tube and K is an unknown constant which cannot be larger than unity. To get a maximum value he placed K equal to unity. To get a minimum value Wiechert assumed that the kinetic energy of the particle was greater than that due to a fall from the cathode to the outer boundary of the dark space. Warburg has shown that this "cathode fall of potential" is independent of the magnitude of the current through the gas, of the pressure of the gas and, within certain limits, of the nature of the electrodes. Since its value in air is about 270 volts, Wiechert assumed a minimum value of 200 volts for KV. Using these assumptions Wiechert found,

$$e/m \ngtr 4 \times 10^7.$$
$$e/m \nless 4 \times 10^6.$$

Wiechert's Direct Measurement of v.

Wiechert[1] measured directly the velocity of the cathode rays, using a method first applied by Des Coudres[2].

The rays are focussed by the curved cathode C (Fig. 3) on the opening in the disc B. A horse shoe magnet D is then brought near the tube so that the beam of rays strikes the disc at a point below its aperture. If a current of very high frequency (in this case it was furnished by a Tesla oscillator and capacities as shown in Fig. 3) is sent through the wire loops MN and M'N' connected in parallel the effect of the current in the loop MN will be to produce a deflection of the beam opposite and equal to that of the magnet. This equality is obtained by adjusting the position of the magnet. At the end of its swing then the beam of rays will pass directly through the opening in the disc B and, if not acted on further, part of them will pass through the disc B' and strike the center of the screen S. If the distance BB' is properly adjusted the rays passing through B when the current is at a maximum will reach B' when the

1 Wiechert, Wied. Ann., 69, p. 739, 1899.

2 Des Coudres, Verhand. d. Physikal. Gesellsch. zu Berlin, 14, p. 86, 1895.

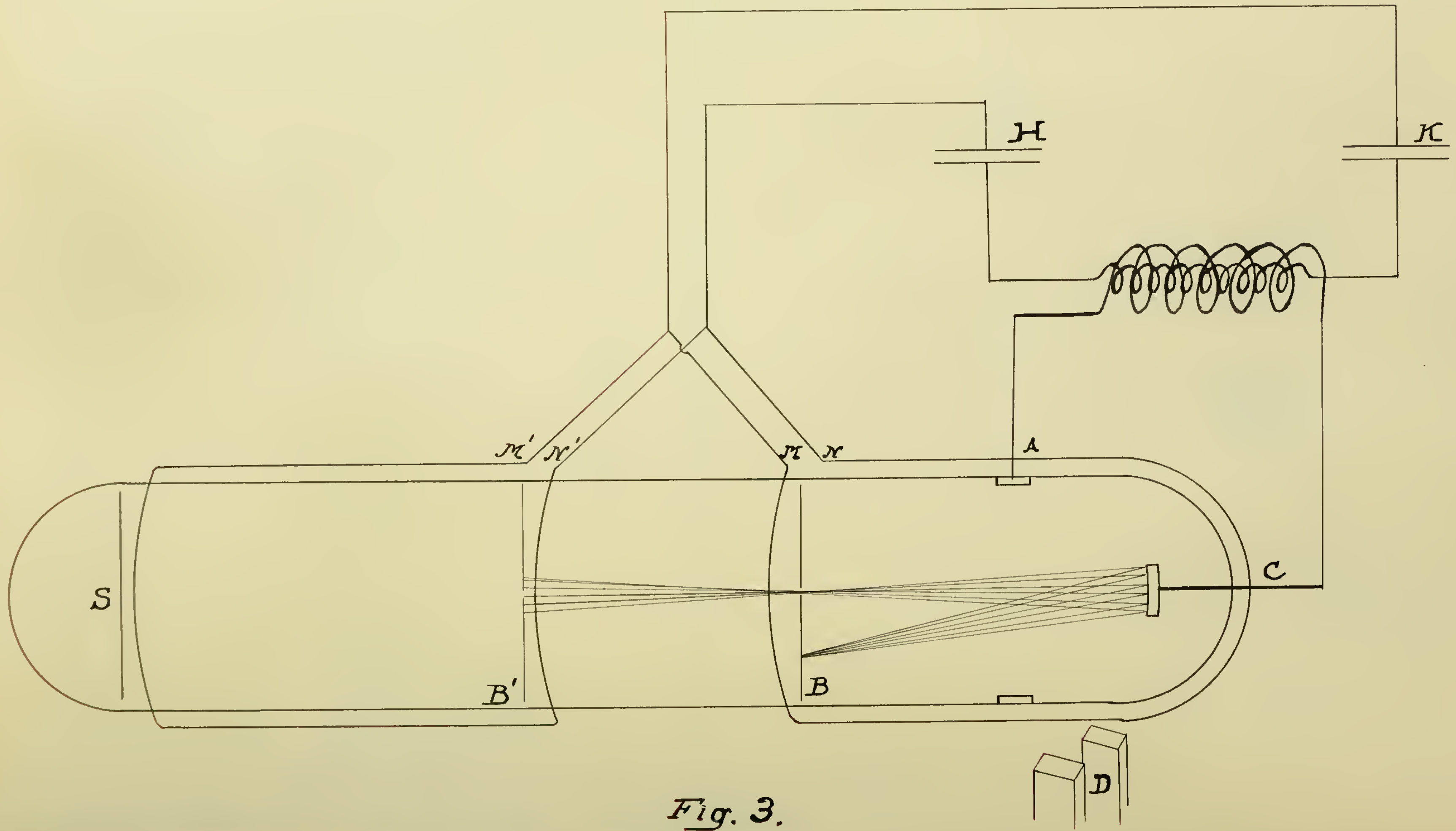



Fig. 3.

current is zero and, not being deflected, will strike the center of the screen. The distance BB' may thus be found when the currents differ by a quarter period, three quarters, etc.

If $\lambda$ is the distance between the circuits when they differ by a quarter period, L is the length of the electric waves passing through the circuits, v the velocity of the cathode rays and V the velocity of light, then;

$$\frac{v}{V} = \frac{\lambda}{L/4},$$

in which all the quantities except v can be measured.

## V.

## METHOD USED IN PRESENT INVESTIGATION.

Since the photographic plate has been used with such great success by Sir J. J. Thomson in England and by Dr. C. T. Knipp in America for recording the deflections of positive particles it was suggested that it might serve also to record the deflections produced by cathode rays. Accordingly the present investigation was undertaken to determine whether the photographic plate was sufficiently sensitive to cathode rays and if so to make a determination of the ratio e/m and the velocity v of cathode rays by recording the deflections produced by the electrostatic and magnetic fields on the photographic plate placed inside the discharge tube.

Probably the most accurate method of determining e/m and v for cathode rays is that making use of the simultaneous deflections of the beam by an electric and a magnetic field. This method was chosen as being the most accurate and if the deflections can be recorded on the photographic plate the measurement of these deflections should also be far more accurate than any measurement that could be made from a willemite screen.

The "current" due to a charge e moving with a

velocity $v$ is evidently $ev$, and the force exerted on this current by a magnetic field of strength $H$ is then $Hev$. Since this force is acting constantly at right angles to the path of the particle the resulting path will be an arc of a circle, $Hev$ being the force toward the center of the circle.

During the time that the particle is passing through an electrostatic field it is subject to a force in a given unchanging direction. The acceleration of the particle in the direction of this force is then proportional to the electrostatic force and the path of the particle is a parabola.

This method is the one used in the present investigation and the formulae giving the values of $e/m$ and $v$ will be derived for this special case, when the electrostatic field exerts its influence during only a part of the path of the particle and the magnetic field is not a uniform field but a field of average strength $H$.

## THE ELECTROSTATIC DEFLECTION.

Suppose the cathode particle to enter through the tube T, as shown in Fig. 4, and to travel in the direction $x$ until deflected by the electrostatic field. If the end of the tube T is just even with the end of the

two electrostatic plates E' and E" then the particle is subject to a force in the y direction as soon as it leaves the end of the tube T but no sooner.

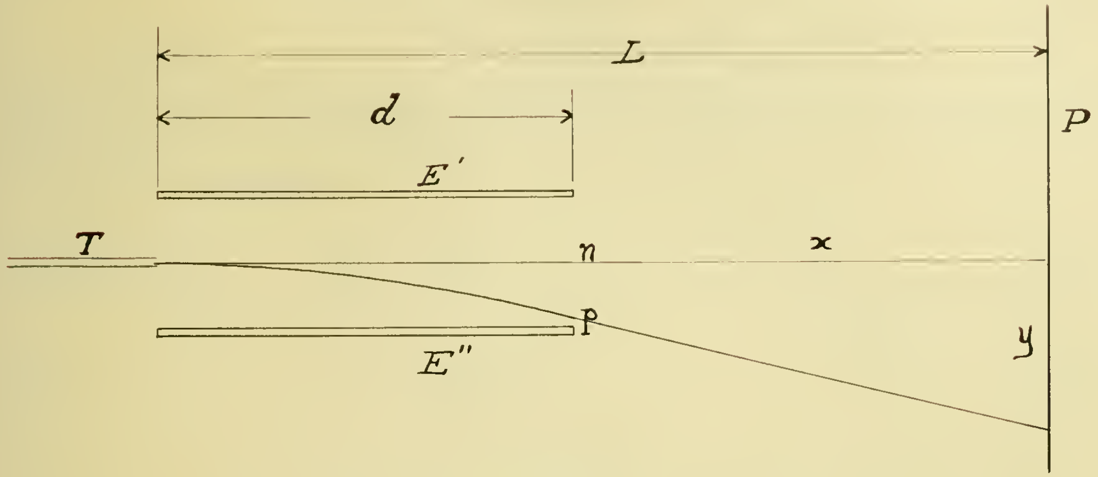



Fig. 4.

Let d be the length of the two electrostatic plates and L the whole distance from the tube T to the photographic plate P used to record the deflections. The deflections produced at the photographic plate may be measured and if the relation between the deflection, the length of the path, the strength of the field, the velocity of the particles and the ratio e/m is known, the values of v and of the ratio e/m may be determined.

The strength X of the electrostatic field between the plates E' and E" is $\frac{\text{volts} \times 10^8}{\text{width in cm.}}$ or $10^8$ times the number of volts per centimeter, in C.G.S units. If e

is the charge on the particle and m the mass of the particle, the force acting on it will be Xe and by Newton's second law

$$Xe = ma = m\frac{d^2y}{dt^2}, \qquad (1)$$

where a is the acceleration in the y direction, parallel to the photographic plate.

Integrating, $$\frac{dy}{dt} = \frac{Xe}{m}t, \qquad (2)$$

and, $$Y' = \frac{Xe}{2m}t^2, \qquad (3)$$

where Y' is the deflection up at the end of the electrostatic plates and t is the time occupied by the particle in traveling the distance d.

Then, $$t^2 = \frac{d^2}{v^2}, \qquad (4)$$

and from (3) $$Y' = \frac{Xed^2}{2mv^2}. \qquad (5)$$

The particle has now been given a velocity V' in the y direction and, since the particle is not subject to a force after leaving the region between the plates, it will travel in a straight line till it reaches the photographic plate.

The velocity V' acquired in the time t will be

$$V' = at = \frac{Xe}{m}t = \frac{Xed}{mv} . \qquad (6)$$

If the time occupied by the particle in traveling the distance L - d is $t' = \frac{L - d}{v}$, then the distance traveled in the y direction after leaving the region between the electrostatic plates will be

$$V't' = \frac{Xed}{mv}\,\frac{(L - d)}{v} = \frac{Xed}{mv^2}(L - d) , \qquad (7)$$

and the whole electrostatic deflection at the photographic plate will be

$$Y = Y' + V't' = \frac{Xed}{2mv^2} + \frac{Xed}{mv^2}(L - d) . \qquad (8)$$

Then,

$$\frac{mv^2}{2} = \frac{X}{Y}\, d(L - \frac{d}{2}), \qquad (9)$$

or,

$$Y = \frac{Xe}{mv^2}\, d(L - \frac{d}{2}) . \qquad (10)$$

The electrostatic deflection is then inversely proportional to the kinetic energy of the particle.

## THE MAGNETIC DEFLECTION.

A charge e moving with a velocity v through a magnetic field of strength H is acted on by a force Hev, and since its path is a circle, Hev is the force toward the center of the circle.

Then, $$Hev = \frac{mv^2}{r}. \tag{11}$$

The curvature of the circle is given by the equation

$$\frac{1}{r} = \frac{\frac{d^2z}{dx^2}}{\left[1 - \left(\frac{dz}{dx}\right)^2\right]^{\frac{3}{2}}}. \tag{12}$$

Since $\left(\frac{dz}{dx}\right)^2$ is small for small deflections it may be neglected. In the apparatus used in this investigation the error due to this neglect, if the deflection is one centimeter or less, is less than .01%.

Then from equations (11) and (12),

$$\frac{d^2z}{dx^2} = \frac{He}{mv}. \tag{13}$$

Integrating, $$\frac{dz}{dx} = \frac{e}{mv}\int_o^x H\,dx. \tag{14}$$

Again, $$Z = \frac{e}{mv}\int_o^L \left[\int_o^x H\,dx\right] dx. \tag{15}$$

Integrate by parts by the formula

$$\int v\,du = uv - \int u\,dv. \tag{16}$$

Let $$v = \int_0^x H\,dx. \tag{17}$$

Let $$dv = H\,dx$$

and $$du = dx.$$

Then, $$u = x = L,$$

and $$\int_0^L \left[\int_0^x H\,dx\right] dx = L\int_0^x H\,dx - \int_0^L Hx\,dx. \tag{18}$$

Combining and substituting in equation (15) we have

$$Z = \frac{e}{mv}\int_0^L (L - x)H\,dx. \tag{19}$$

Hence, $$\int_0^L (L - x)H\,dx \tag{20}$$

is the quantity to be determined. Since both H and x are variables this cannot be integrated and a method must be found to measure it graphically or to measure some quantity which is equal to the integral.

If L is the altitude of a right angle triangle and D is its base its area is

$$\frac{D}{L}\int_0^L (L - x)\,dx$$

where L is measured in the x direction. Since this integral is so near like that above it occurred to Sir J. J. Thomson[1] that a method might be devised making use of this integral to determine the value of the unknown integral.

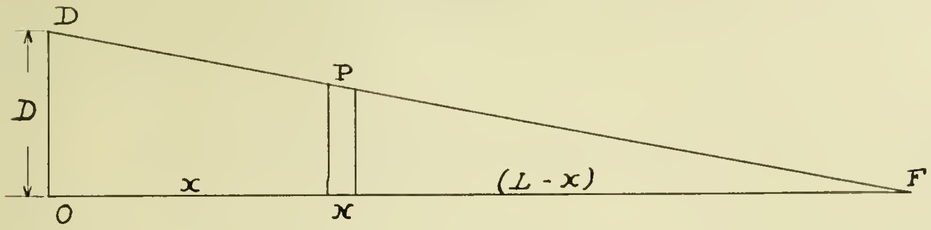



Fig. 5.

Construct a triangular frame, as shown in Fig. 5, whose base D is small enough that the field H is constant over any range parallel to the base and whose altitude is the distance from the tube T to the photographic plate. Wind around this frame, in the plane DOF, n turns of wire. If the triangle is placed with its base at the source of rays and its vertex at the photographic plate the field H will be constant over the area PN dx. The magnetic induction through this area will then be

$$I = n\int_{0}^{L} PN\ H\ dx. \qquad (21)$$

1 J. J. Thomson, Phil. Mag., XVIII., p. 844, Dec. 1909.

By similar triangles

$$\frac{PN}{D} = \frac{(L - x)}{L}, \tag{22}$$

and

$$PN = \frac{D}{L}(L - x). \tag{23}$$

Placing this value of PN in equation (21) we obtain the value for the total magnetic induction I through the test coil,

$$I = \frac{nD}{L}\int_0^L (L - x)H\,dx. \tag{24}$$

Hence,

$$\int_0^L (L - x)H\,dx = \frac{IL}{nD} \tag{25}$$

and this value of the integral may be placed in (19) giving

$$Z = \frac{eIL}{mvnD}, \tag{26}$$

which is the magnetic deflection.

The value of the total magnetic induction may be measured by means of a Grassot fluxmeter or by a ballistic galvanometer.

From (26),

$$\frac{e}{mv} = \frac{nDZ}{IL}. \tag{27}$$

Having obtained values for $e/mv$ and $mv^2/e$, by combining equations (9) and (27) either $e/m$ or $v$ may be eliminated.

Multiplying (9) by (27),

$$v = \frac{XZ}{YI}\frac{nDd}{L}(L - \frac{d}{2}). \qquad (28)$$

Multiplying (27) by (28),

$$\frac{e}{m} = \frac{XZ^2}{YI^2}\frac{n^2D^2d}{L^2}(L - \frac{d}{2}). \qquad (29)$$

Equation (28) may be expressed as

$$v = \frac{XZ}{YI} A \qquad (30)$$

where $$A = \frac{nDd}{L}(L - \frac{d}{2}). \qquad (31)$$

The constant A depends on the dimensions and number of turns of wire on the test coil, the length of the electrostatic plates and the length of the path of the undeflected beam. Similarly equation (29) may be expressed as

$$\frac{e}{m} = \frac{XZ^2}{YI^2} B \qquad (32)$$

where $$B = \frac{n^2D^2d}{L^2}(L - \frac{d}{2}). \qquad (33)$$

The constant B also depends on the dimensions of the discharge tube, the electrostatic plates and the test coil.

## VI.

## DESCRIPTION OF APPARATUS.

The neck was cut from a two liter boiling flask and a hole about 4 centimeters in diameter was blown in the flask opposite the neck. The aluminum cathode, shown at C in Fig. 6, had its aluminum supporting rod encased in a small glass tube which at the joint B was joined to a larger tube, a bulb being blown in this larger tube to make the tight joint at B. This joint was secured with red wax. By warming the wax the cathode position could be altered, and so adjusted that the beam passed through the tube T, without letting down the vacuum. The cathode and its glass casing entered the discharge tube through a tube D, of about 2 centimeters diameter, which was fastened with red wax to the short neck of the flask. Side connections were blown for connection to the Gaede pump and charcoal tubes. The ring anode was carried by the part of the tube extending inside the discharge tube. The cathode extended a little beyond the end of this tube D and was placed nearly in the center of the discharge tube, the anode and cathode being about 8 centimeters apart.

The vessel was exhausted by the Gaede pump to

a pressure of about .001 cm. and the application of liquid air to the charcoal tubes then brought the pressure down to .0003 cm. or less. A bulb containing phosphorous pentoxide was placed in the pump connection, as near as possible to the discharge tube, and whenever convenient the vessel was permitted to stand, partly exhausted, for a day or two to remove all traces of moisture.

When any leaks occurred in the red wax joints the joints were painted over with the soft wax made of equal parts of bees wax and resin. The joints were finally made sufficiently tight so that when left with a pressure of .002 cm. for four days the cathode beam was still easily visible, the pressure not having raised above .0025 cm. The pressure was measured by a McLeod gauge included in the pump connection. The bulb of the gauge had a volume of 100 cubic centimeters and gave, under compression to .1cc. the ratio of 1 to 1000.

The cathode beam was cut down to a small pencil of rays by passing through the copper tube T, of about .04 cm. diameter and 3 cm. length. The end of the copper tube was placed just at the end of the two electrostatic plates and the tube surrounded by an iron tube so that little electrostatic or magnetic force was exerted on the particles until they left the tube T. After leaving the tube T

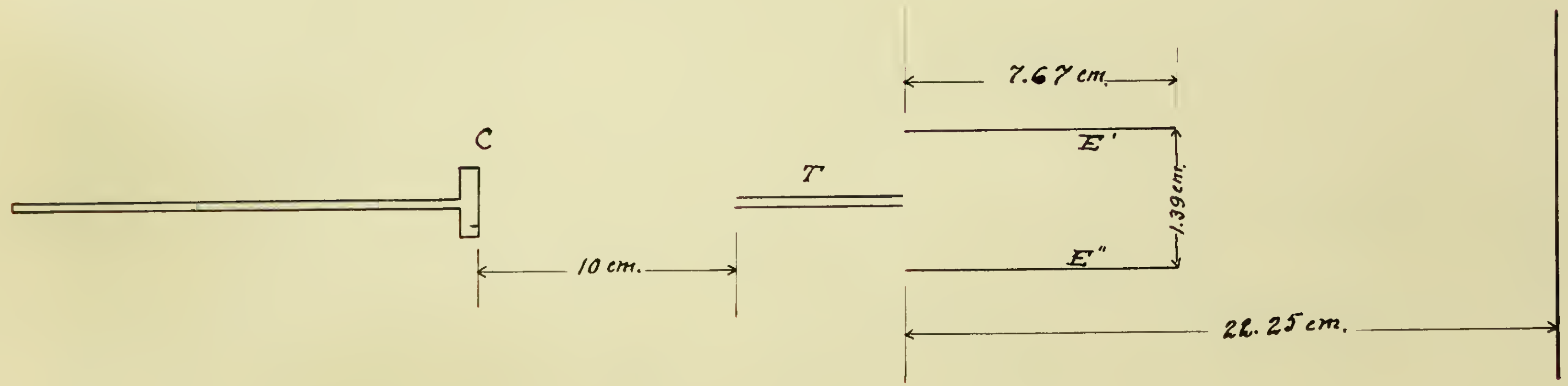



Fig. 7.

the rays passed between two aluminum electrostatic plates E' and E" and then on to the photographic plate. The photographic plate P was cut to a circle and placed in the circular plate holder H which could be slid back and forth in the brass cylinder C. The brass cylinder was about 8 inches long and made from a piece of six inch brass tubing whose walls were about a quarter of an inch thick. The tube was bored out until its walls were about half as thick and a shorter piece was turned down until it slipped easily inside the longer tube. This shorter piece then formed the plate holder. A facing ring was soldered to the holder against which the photographic plate could rest. A coil spring held a circular brass plate against the back of the photographic plate so that the plate could not shift once it was placed in the tube. A brass plate about half an inch thick was soldered to the brass cylinder C and a hole about 2 inches in diameter was bored through its center. This opening was then threaded to fit the hard rubber tube R which carried at its other end the iron tube T. The base plate of the brass cylinder C was also threaded to screw into a brass ring I which was fastened with red wax to the plate glass base K. Waxed to this plate glass base, and surrounding the brass cylinder C, was a glass cylinder of about eight inches diameter made by

cutting the top and bottom from a large bottle. The other end of this glass cylinder was closed by a glass plate G similar to the one carrying the brass ring I. The plate glass base K had a hole two inches in diameter bored in its center and one end of a glass tube M, to the inside of which was fastened - also with red wax - the hard rubber tube R, was fastened to the plate so that the threaded end of the hard rubber tube screwed into the threaded hole of the brass base of the cylinder C; the tube M was swelled slightly to prevent its being forced through the plate K when under vacuum. The other end of the tube M was fastened with red wax to the discharge tube which, as before mentioned, was made from a two liter flask. When the brass cylinder C, carrying the plate holder H, was screwed into the ring I the only chance for light to enter the chamber containing the photographic plate was through the tube T. This tube was so small and so long that the amount of light reaching the photographic plate through it was not enough to affect the plate.

The two aluminum electrostatic plates E' and E" were mounted in slots cut in the sides of the hard rubber tube R. They were connected to the high potential storage battery by two fine wires run out through the sides of the hard rubber tube R and the glass tube M. Considerable

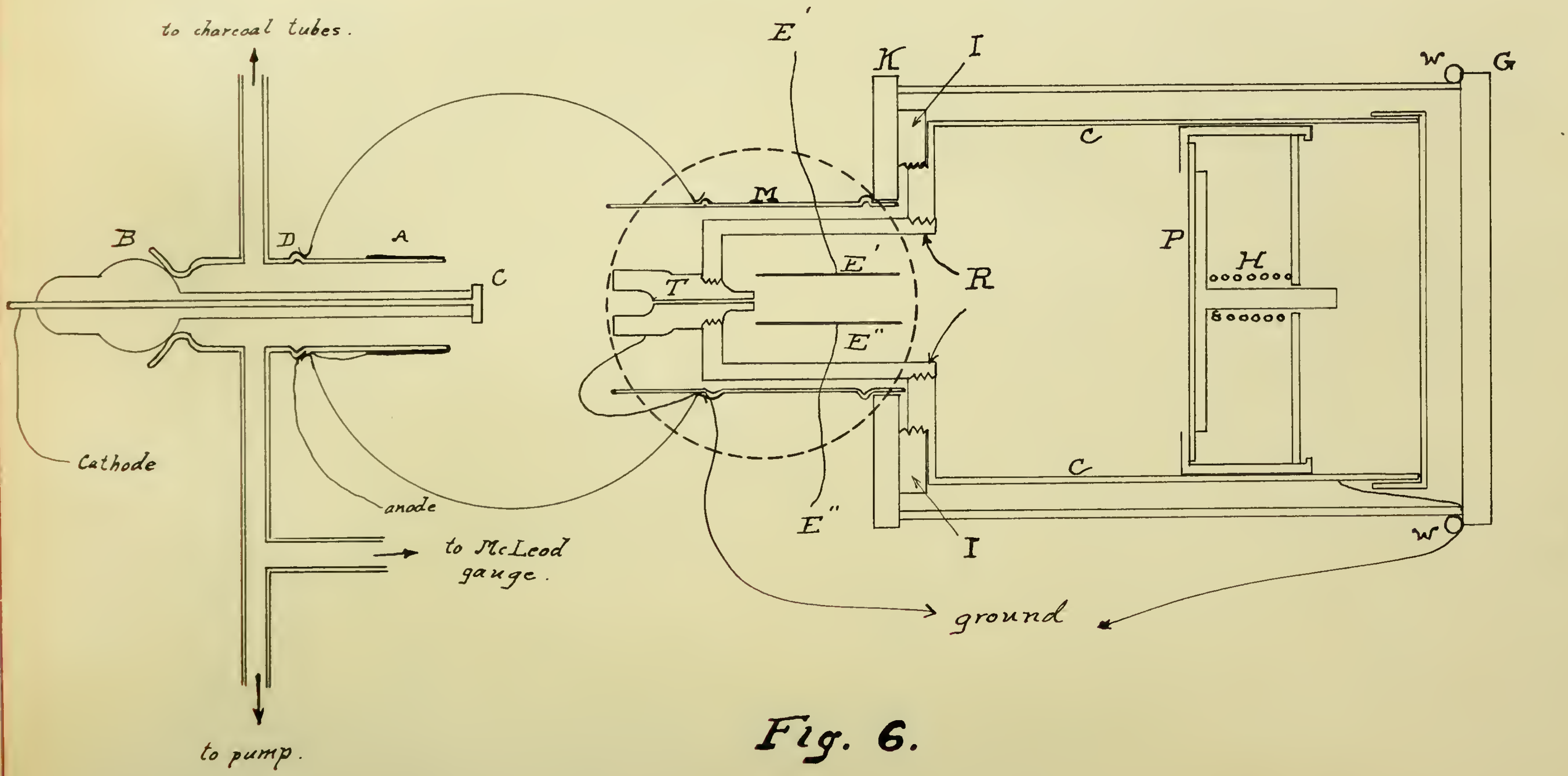

to charcoal tubes.
B
D
A
C
Cathode
anode
to McLeod gauge.
to pump.
E'
E''
K
I
M
T
R
P
H
C
W
G
ground

Fig. 6.

difficulty was experienced in getting the electrostatic plates accurately parallel and the ends of the two exactly even with the end of the tube T. They were, however, finally mounted so that their distance part was uniform.

The glass plate C was fastened on with a soft wax made of bees wax and resin and could be removed by circulating hot water through the heating coil W. The heating coil W was made by bending a glass tube of about 4 millimeters internal diameter into a circle and joining the ends. A branch was then sealed in at each side and the two branches connected to the water heater. The heater consisted of a piece of glass tubing of about two centimeters internal diameter and about thirty centimeters length. It was held at an angle of about 45° and the two branches of the heating coil connected to its ends by short pieces of rubber tubing. The water was heated by a bunsen burner placed under the glass heater, the flame not quite reaching the glass. The heating coil W would be a great deal more satisfactory if made from brass tubing, since it would heat with less danger of breaking and would be much easier to construct. The hot water heater could also be made of metal.

The solenoid used to produce the magnetic field

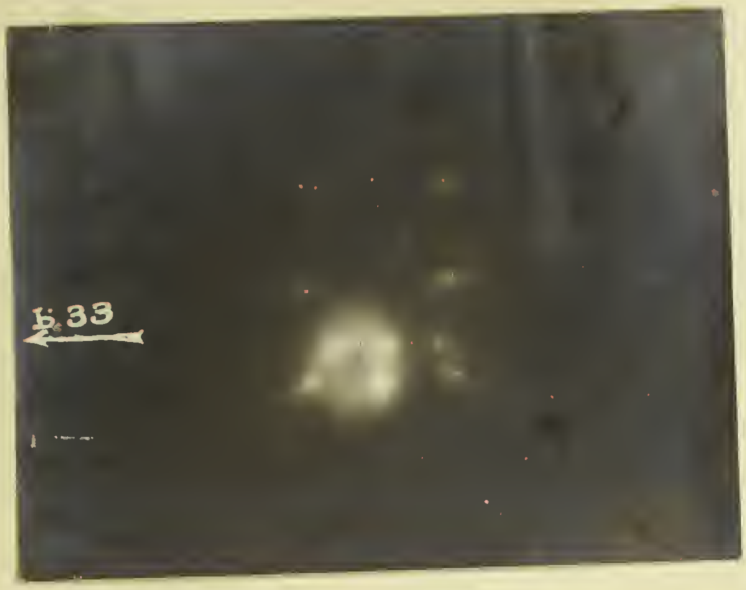
is shown by the dotted circle. It was wound in two parts, each part consisting of seven layers of number 16 copper wire with 80 turns in each layer. After the rest of the apparatus had been set in position the two parts of the solenoid were brought up on each side of the tube M and connected in series.

The tube T and the brass cylinder C were connected to earth by wires led out through the wax joints. To lead the wire out through the soft wax joint at the plate G a small notch was cut, with a file, in the glass cylinder and the wire lay in this notch. No difficulty was experienced with leakage at this point. After the parts were well grounded no trouble was caused by fogging of the plates.

For ease in making the adjustments a willemite screen, made by depositing willemite in alcohol, was made on the plate G closing the end of the exhausted vessel. The screen was ruled into half centimeter squares and approximate deflections could be read off.

The rays were produced by the discharge from an induction coil operating with an attracted hammer interrupter. The varying potential of the coil caused the spot of light on the plate to stretch out into a strip

when deflected by either field. The definition was increased in the higher vacua and this lengthening effect was greatly reduced by covering the outside of the discharge tube with strips of tin foil and grounding them. In the best photographs obtained the spot of light was still slightly lengthened, being anout one and one half times as long as broad. With the induction coil operating on 20 volts the time required for the beam to affect the photographic plate was about ten minutes. The plates were exposed to each of the fields acting alone, both direct and reversed, and to the simultaneous action of the two fields. This, with the position of the undeflected beam, gave nine spots on each plate, arranged at the corners and middles of the sides of a rectangle with the undeflected spot in the center. This is shown in the accompanying photograph (33 Fig. 8) which was printed from the last plate exposed. The horizontal are the electrostatic and the vertical the magnetic deflections. In this particular plate there are eight spots showing at the corners and middles of the sides of a rectangle; the other three spots visible form the end of a smaller rectangle. The large blurred spot is due to the faulty washing of the negative. The central spot and the other five spots of the inner rectangle failed to appear.



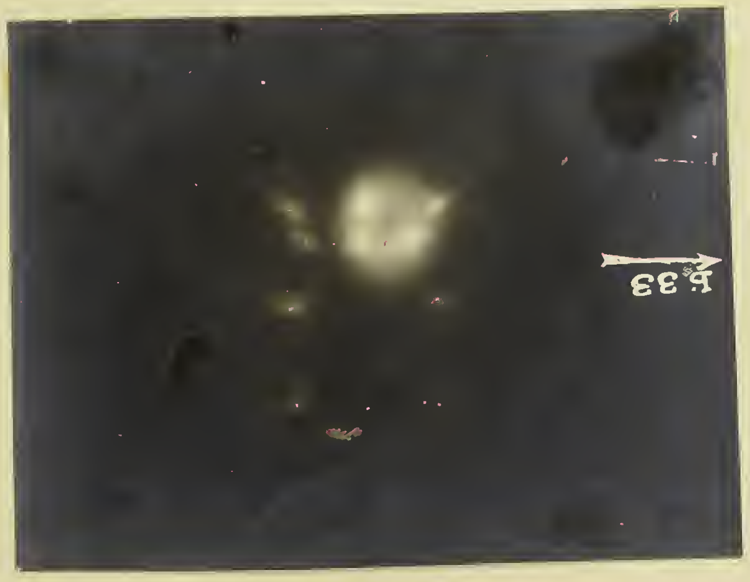



Fig. 8.

The magnetic and electrostatic fields were kept constant during the exposure of the eight spots forming the outer rectangle and again constant, at smaller values, during the exposure of the inner rectangle. When all the spots exposed show it is possible to make two determinations of $e/m$ and $v$ from each plate (one from each rectangle) and in each determination the electrostatic and magnetic deflections are the means of three double deflections. The two values of $e/m$ obtained from this plate agreed within about 2 per cent.

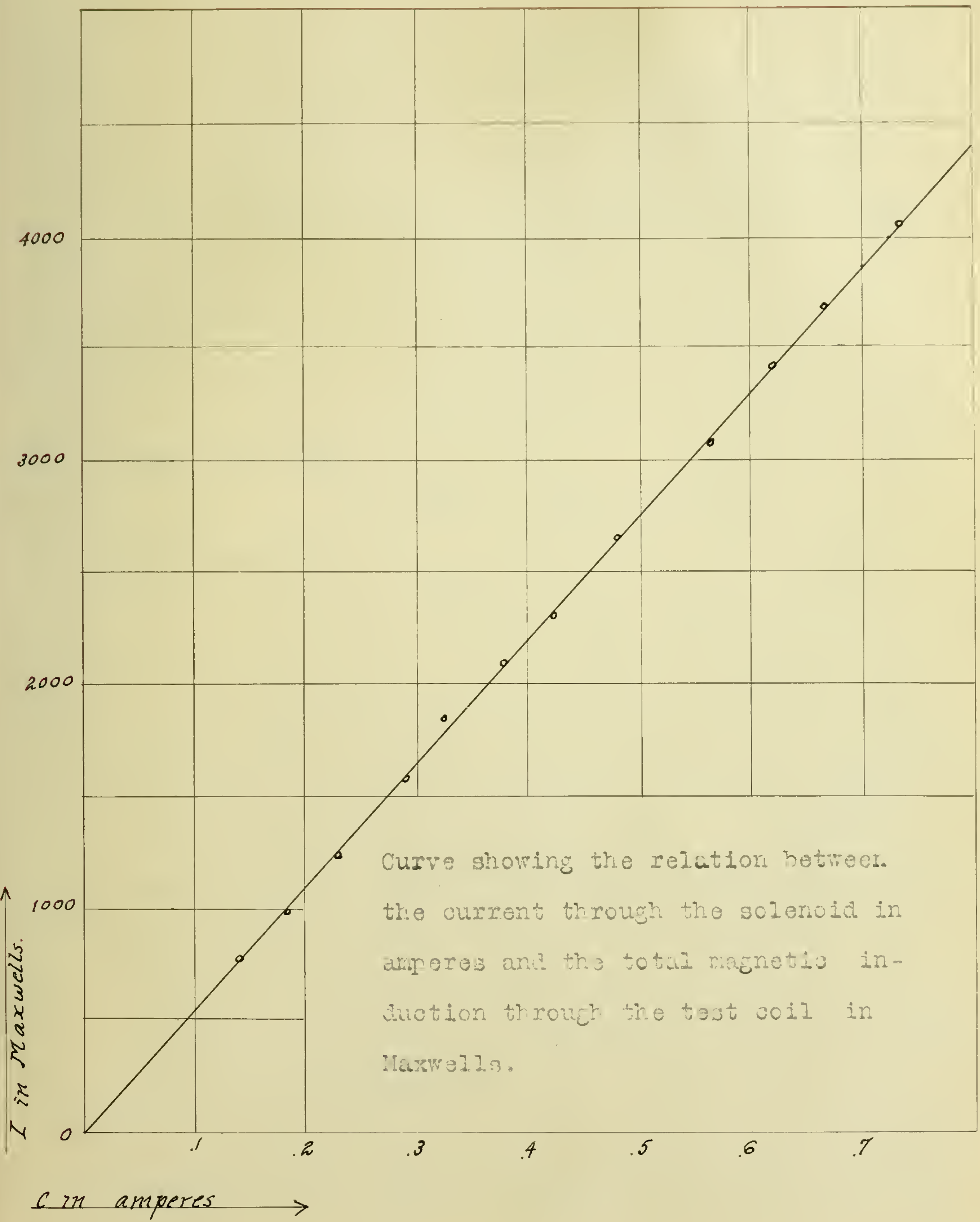
With the small induction coil the time of exposure was so long that some difficulty was experienced in keeping all factors constant. A larger coil operating on 110 volts and with a Wehnelt interrupter produced the spots on the plate with an exposure of only two seconds, but the discharge through the tube was so terrific that the pressure doubled in that time, due to gases given off by the wax with which the joints were made. If the discharge tube were made with few or no wax joints or the joints so protected that tha rays could not strike the wax the charcoal tubes should be able to handle the gases given off.

The potential difference of the two electrostatic plates was maintained by a high potential storage

battery, the difference of potential being measured by a Weston voltmeter.

The current through the solenoid producing the magnetic field was furnished by a few very constant storage cells and was measured by a Siemens & Halske ammeter reading from 0 to 3 amperes. The total magnetic induction through the test coil was measured with the current varying from .2 ampere to 8 amperes and a curve plotted as shown in Fig. 9, using the current in amperes as abscissas and the total induction in Maxwells as ordinates. The total induction varied from 1400 Maxwells for .2 ampere to 4000 Maxwells for .65 ampere. The values of the total induction for the larger currents were divided by 10 and the current by the same amount and the results plotted. Since these points also lay on the straight line curve it is permissable to find by extrapolation the values of the total induction for currents smaller than .2 ampere.

The distance from the tube T to the photographic platewas measured with a cathetometer and the length and distance apart of the two electrostatic plates by a vernier caliper.




Curve showing the relation between the current through the solenoid in amperes and the total magnetic induction through the test coil in Maxwells.

## Constants of Apparatus.

The various constants of the apparatus were determined and are here recorded under the letters used to denote them in the derivation of the formulae.

$$n = 50 \text{ turns.}$$

$$D = 2.11 \text{ cm.}$$

$$d = 7.67 \text{ cm.}$$

$$L = 22.25 \text{ cm.}$$

Then, from (31),

$$A = \frac{nDd}{L}\left(L - \frac{d}{2}\right) = 669.2,$$

and, from (33),

$$B = \frac{n^2D^2d}{L^2}\left(L - \frac{d}{2}\right) = 3174.6 \ .$$

The results given in tables I and II are calculated from the plates in the order of their exposure. The first fourteen plates failed to show spots well enough defined that measurements could be made from them. In plates 15 to 23 the potential difference of the electrostatic plates was taken as the rated E.M.F. of the storage cells. This was later found to be in error by nearly ten per cent. In these plates too, the electrostatic deflection was too large to permit the neglect of the stray electric field lying beyond the ends of the two plates. The formula as developed is applicable only

when this stray field is neglected. The results from plates 15 to 25 are therefore discarded. The values from plates 26 and 33a are also cast out because the values are obtained from measurements on only three spots while on the other plates six or more spots showed, thus making each deflection the mean of at least two double deflections. The double deflection is measured from the spot with the field direct to that with the field reversed. The values of e/m and v are calculated from deflections read on the willemite screen but these were merely for a check and hence are also not included in the averages. There remain then the plates listed in table II. from which the values of e/m and v may be taken. The average from these plates gives

$$e/m = 1.91 \times 10^7.$$

$$v = 4.378 \times 10^9.$$

The most probable value of e/m is about 1.76 x $10^7$. Values very near this have been found by Seitz, by Kaufmann and by Simon.

Table I.

| Pla-te. | Press. cm. | cm. Y | cm. Z | amp. C | Maxw. I | V/d X | $e/m \times 10^7$ | $v \times 10^9$ |
|---|---|---|---|---|---|---|---|---|
| 15 | .0003 | 1.575 | 2.56 | .725 | 4470 | 288 | 1.904 | 7.007 |
| 16 | .0003 | 2.17 | 4.15 | .704 | 4300 | 288 | 3.925 | 8.571 |
| 20 | .0005 | 3.2 | 3.28 | .600 | 3700 | 450 | 3.517 | 8.342 |
| 21 | .0015 | 3.87 | 3.07 | .596 | 3680 | 450 | 2.570 | 6.491 |
| 23 | .002 | 2.98 | 3.06 | .596 | 3680 | 288 | 2.121 | 5.377 |
| 26 | .002 | 1.80 | 2.20 | .490 | 3020 | 144 | 1.348 | 3.900 |
| scr. | | 2.66 | 2.21 | .297 | 1830 | 131 | 2.343 | 4.779 |
| 33a | .0004 | .700 | .830 | .155 | 952 | 59.7 | 2.058 | 4.976 |

Table II.

| Pla-te. | Press. cm. | cm. Y | cm. Z | amp. C | Maxw. I | V/d X | $e/m \times 10^7$ | $v \times 10^9$ |
|---|---|---|---|---|---|---|---|---|
| 24 | .0015 | 2.52 | 2.56 | .598 | 3690 | 273.3 | 1.658 | 5.035 |
| 27 | .001 | 3.75 | 1.20 | .208 | 1270 | 196.5 | 1.485 | 3.550 |
| 28 | .0005 | 1.92 | 1.50 | .250 | 1540 | 133 | 2.087 | 4.515 |
| 29 | .0004 | 1.24 | 1.20 | .258 | 1590 | 133 | 1.840 | 5.416 |
| 30 | .0015 | 1.73 | 1.60 | .267 | 1640 | 133 | 2.361 | 5.019 |
| 32 | .00025 | .597 | .466 | .0995 | 616 | 60.4 | 1.838 | 5.122 |
| 32 | .00025 | .937 | .725 | .150 | 928 | 90.3 | 1.867 | 5.038 |
| 33b | .001 | .966 | 1.312 | .250 | 1537 | 89.5 | 2.145 | 5.298 |
| | | | | | average | | 1.910 | 4.878 |

# VII.

## SUMMARY.

1. The photographic plate may be successfully used to record the deflections of a cathode beam. With a very heavy discharge through the tube well defined negatives were obtained with exposures as short as two seconds.

2. The value of the ratio of the charge to the mass, as determined by this investigation, is $1.91 \times 10^7$. The value of the velocity v is $4.878 \times 10^9$ centimeters per second.

Some modifications are being made in the form of the apparatus and it is expected that under more favorable conditions a very accurate value of the ratio e/m may be determined.

In conclusion I wish to express my appreciation to Professor A.P.Carman for the facilities so kindly placed at my disposal, and to Dr. C. T. Knipp for his many suggestions.

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May 30, 1912.